Subsequence Counting

This is a **regular task**. You must submit a PDF, which can be produced using the LaTeX template on Moodle, exported from a word processor, hand-written or any other method.

Let $A = a_1 \dots a_n$ and $B = b_1 \dots b_m$ be two strings of length n and m respectively. We say that a string A occurs as a subsequence of another string B if we can obtain A by deleting some of the letters of B.

We want to count the number of times of A appears as a subsequence B. For example, suppose

$$A = \texttt{pass}$$
$$B = \texttt{please, no more data structures}.$$

Then A occurs as a subsequence of B precisely five times:

- please, no more data structures
- please, no more data structures
- please, no more data structures
- please, no more data structures
- please, no more data structures

(a) This problem is similar to 3.07 Subsequence Search. Briefly explain why a similar greedy approach will not work here.

(b) Design an algorithm that runs in $O(nm)$ time to count the number of occurrences of A as a subsequence of B.

Rubric.

(a) Explain why a similar greedy method will not work for the given problem. Expected response length: one sentence.

(b)
- Design a dynamic programming algorithm that solves the problem in English and mathematical notation (not code or pseudocode).
- Include the subproblem definition, recurrence, base case(s), order of computation, final answer calculation, and any other steps required for your algorithm.
- Justify your recurrence, base case and final answer.
- Analyse the time complexity of your algorithm.

Expected length: about half a page.